Phyllotaxis and Fibonacci

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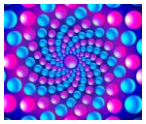


phyllotaxis – the spiral pattern arrangement of organs on plants

It is said that Leonardo Pisano's boyhood friends in Pisa, Italy, nicknamed him 'The Blockhead.' But the nickname that has lasted since the Middle Ages is Fibonacci (fē-bə-ˈnä-chē). This nickname came from either a short version of 'filius Bonacci' (son of Bonaccio) or a translation of 'lucky son.' Today the name Fibonacci mostly just means, 'an amazing mediaeval European mathematician who found a cool mathematical explanation for a tricky natural mystery.' Definitely not a blockhead, right?

In 1202, Fibonacci published *Liber Abaci* (*Book of Calculation)* to help Europeans learn the Arabic system of math which he preferred. The book included Fibonacci's solution to a puzzle that had stumped mathematicians and scientists for centuries. This puzzle focused on predicting the number of bunnies born in one year to a family started by two bunnies having 1 male and 1 female baby bunny per month.

Bunches of bunnies later, Fibonacci discovered a number sequence to describe the bunnies' big family tree:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987…

"Well, duh" Moment	As each month passed more bunnies were giving birth than just the first mama & papa. So each numeral in the sequence shows the number of bunnies at the end of each month.
"A-ha!" Moment	Each numeral in the sequence also is the sum of the two preceding numerals.

Go ahead, prove it to yourself:

0 + 1 = _____ 1 + 1 = _____ 1 + 2 = _____ 2 + 3 = _____

21 + 34 = _____ 34 + 55 = _____ 55 + 89 = _____ 89 + 144 = _____

What will the next number in the sequence (as shown above) be? _____

For a clever, quick overview of spirals and Fibonacci numbers, check out parts 1 & 2 of Vi Hart's video on Khan Academy, ***Doodling in math: Spirals, Fibonacci, and being a plant*** (~6 minutes each); save part 3 for later.

Also watch, ***Open letter to Nickelodeon, re: SpongeBob's pineapple under the sea*** (~2 minutes).

Free Video URL: http://www.khanacademy.org/math/recreational-math/vi-hart/spirals-fibonacci/v/doodling-in-math--spirals--fibonacci--and-being-a-plant--1-of-3. Or just try: http://tinyurl.com/ViHartVideos .

Now that you have the big picture, use the next pages to dig into some finer points.

SEEDS - Sunflower faces display two sets of spiraling seeds going in opposite directions. Most sunflowers show seed spirals with 34 and 55 seeds. Small sunflower seed spirals use 21 and 34 seeds. The biggest sunflowers have 89 and 144 seeds per spiral.

1. *Complete:* The connection between number of seeds in the sunflower spirals and Fibonacci's sequence is…

_____

_____

2. Review this gallery of Fibonacci samples: http://www.math.smith.edu/phyllo/Gallery/Pages/FrameSet.htm

   Which plant do you think has the most interesting spiral patterns? _____

3. Find three fill-in-the-blank answers as you take the virtual tour of the exhibit 'Plant Spirals: Beauty You Can Count On' http://www.math.smith.edu/phyllo/EXPO/ExpoIntro.html:

   a. The 'short apical meristem' of a plant is _____.

   b. _____ are the early-stages of a plant's growth units (e.g. leaves, petals). The next or newest one will grow in the _____ _____ location.

   c. The measurement between a plant's parts that show Fibonacci spirals is the _____.

4. Review http://www.math.smith.edu/phyllo/About/Classification.html to see the four main classifications of spirals. Draw and label them here:

5. See http://www.math.smith.edu/phyllo/About/Lattices/SpiralLattices.html

   Parastichies are _____
   They occur when _____

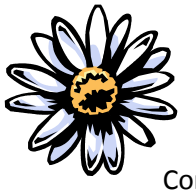
   _____

NEEDLES & CONES - There usually are 2, 3 or 5 needles in each cluster of pine needles. The special leaves that make pine cones also form spirals that tend to follow Fibonacci.

Check out these pine cone spiral images: http://www.maths.surrey.ac.uk/hosted-sites/R.Knott/Fibonacci/fibnat.html#pinecones and then answer these questions:

1. Pine cones usually have _____ sets of spirals.
2. In the pinecone on the right (at the link above):
   a. How many rows are in yellow-pink-red spiral? _____ In the blue-green spiral? _____
   b. Is this Fibonacci? Y / N Why? _____

      _____



PETALS

Depending on variety or growing conditions, daisies usually have 21, 34, 55, or 89 petals.
Find an image of your state flower here http://www.statesymbolsusa.org/Lists/state_flowers.html.

Complete this chart to see if your state flower follows Fibonacci, too.

State:	Fibonacci? Y / N Explain:
Flower:	
Number of petals:	

LEAVES follow Fibonacci's sequence as they alternate their way up stems. This leaf placement can be defined by a phyllotactic ratio. In simple terms, the phyllotactic ratio is whatever fraction of a circle you can see between one leaf and the next.

1. Review leaf arrangements with Fibonacci here: http://www.maths.surrey.ac.uk/hosted-sites/R.Knott/Fibonacci/fibnat.html#plants. The mathematical patterns are cool, but how does the arrangement of leaves on the stem actually help the plant?

_____

_____

2. Click through the photos http://theobrominated.blogspot.com/2011/06/she-loves-me-she-loves-me-not.html Review this discussion (including some advanced biological terms) of leaf and petal spirals.
   a. Define *monostichous* - _____
      _____
   b. Define *distichous* - _____
      _____

3. Collect at least one stem with leaves. Draw and label your sample below. Labeling can include plant name, location, scale/actual dimensions, monostichous or distichous.

Classroom Cultivars for

Phyllotaxis and Fibonacci

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A variety of follow up or classroom activities for cultivating more mastery of fascinating math and science phenomena.

Suitable for flipped biology & math classrooms.

Aligns with:
NGSS Crosscutting Concepts: (1) Patterns; (2) Scale, proportion, quantity; (3) Structure and function
NGSS Science & Engineering Practices (5) Using mathematics and computational thinking
CCSS-Math: Linear and Exponential Relationships/Functions (multiple grades)
ASPB Principles of Plant Biology: #s 6, 7, 11
www.aspb.org/12principles

1. Define *phyllotaxis*:
   - As an individual task/quiz
   - With a labeled doodle
   - By class consensus
   - As a tweet (140 characters or less)
   - In a couplet, triplet or limerick

2. Review (actually re-view or just discuss) parts 1 & 2 of *Doodling in math: Spirals, Fibonacci, and being a plant.*
   a. Clarify basic concepts and misconceptions related to the science and math.
   b. Address the fun and funky format and tone. Is it 'okay' for math and science to be portrayed like this?

3. Make *phi-angle-atrons* (pause Hart's videos for directions) and create colorful cone, flower or leaf diagrams.

4. Have students share the plant stem samples and spiral counts they did at home. Offer flower and cone samples and allow for more counting and spiral searching.
   a. Mark the spirals with tape, colored glue/paint, or other supplies.
   b. Analyze the non-Fibonacci samples. Does anyone see spirals or a pattern of any sort? (Spoiler alert: those who watched part 3 of Hart's video could skew the open-ended nature of this discussion).
   c. Categorize the samples as Fibonacci and non-Fibonacci.
   d. Generate hypotheses for the non-Fibonacci samples.

5. Watch part 3 of *Doodling in math: Spirals, Fibonacci, and being a plant* (~6 minutes) http://www.khanacademy.org/math/recreational-math/vi-hart/spirals-fibonacci/v/doodling-in-math--spirals--fibonacci--and-being-a-plant--part-3-of-3
   a. Define, compare, and contrast Lucas and Fibonacci Numbers.
   b. Analyze samples from #4 to find any that show Lucas Numbers.

6. What is phi, really? Ask students to add examples to those Meera Kurup posted on Vi Hart's video page:

It is an irrational number equal to half of one more than square root of five.
It is also the solution to the quadratic equation: 1 + x = x^2
It is also equal to twice the sine of 54 degrees.
It is the ratio between A and B such that A/B = (A+B)/A where A is greater than B.
It is a non - terminating number. It is, you can say: 1.61803398874989...
It is the ratio of a diagonal of a regular pentagon to its sides.
It is related to gender distribution in bees... Flower arrangements... Formation of hermit crab shells and more!

7. *Ask students to…* Consider all you've seen and learned to create (with assigned or self-selected materials) an artistic image that addresses the aspects of plant biology and Fibonacci that inspire YOU.
   - Review 'Art Meets Math: A Visual Relationship Between Art and Mathematics' http://owbirdsall.com/art-meets-math/ for ideas, as needed.
   - Include a title and brief paragraph to explain the art-math-biology connections featured in your art.

8. AP Thinking: Share the class-appropriate slides and notes to help guide your students through ***Genetic Control of Leaf Development*** http://www.plantcell.org/site/teachingtools/TTPB3.xhtml.

ANSWERS for Phyllotaxis and Fibonacci

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987…

Go ahead, prove it to yourself:

0 + 1 = *1*; 1 + 1 = *2*; 1 + 2 = *3*; 2 + 3 = *5*; 21 + 34 = *55*; 34 + 55 = *89*; 55 + 89 = *144*; 89 + 144 = *233*

What will the next number in the sequence (as shown above) be? *1597*

SEEDS - Sunflower faces display two sets of spiraling seeds going in opposite directions. Most sunflowers show seed spirals with 34 and 55 seeds. Small sunflower seed spirals use 21 and 34 seeds. The biggest sunflowers have 89 and 144 seeds per spiral.

6. *Complete:* The connection between number of seeds in the sunflower spirals and Fibonacci's sequence is…*the number of seeds in each spiral pair are consecutive numbers in the Fibonacci sequence*.
7. Review this gallery of Fibonacci samples: http://www.math.smith.edu/phyllo/Gallery/Pages/FrameSet.htm Which plant do you think has the most interesting spiral patterns? [*individual choice*]
8. Find three fill-in-the-blank answers as you take the virtual tour of the exhibit 'Plant Spirals: Beauty You Can Count On' http://www.math.smith.edu/phyllo/EXPO/ExpoIntro.html:
   a. The 'short apical meristem' of a plant is *the tip of the plant where growth begins*.
   b. *Primodia* are the early-stages of a plant's growth units (e.g. leaves, petals). The next or newest one will grow in the *least crowded adjacent* location.
   c. The measurement between a plant's parts that show Fibonacci spirals is the *Golden Angle, 137.5^0*.
9. Review http://www.math.smith.edu/phyllo/About/Classification.html to see the four main classifications of spirals. Draw and label them here:

distichous	*whorled*	*spiral*	*multijugate*

10. See http://www.math.smith.edu/phyllo/About/Lattices/SpiralLattices.html
    Parastichies are *the visible spirals in a spiral lattice.* They occur when *the plant's primordia grow to fill the next available space and the observer connects the nearest points into spirals.*

NEEDLES & CONES - Check out these pine cone spiral images: http://www.maths.surrey.ac.uk/hosted-sites/R.Knott/Fibonacci/fibnat.html#pinecones and then answer these questions:

3. Pine cones usually have *two* sets of spirals.
4. In the pinecone on the right (at the link above):
   a. How many rows are in yellow-pink-red spiral? *5 -five* In the blue-green spiral? *8 -eight*
   b. Is this Fibonacci? Y / N Why? YES – *5 and 8 are consecutive numbers in the Fibonacci sequence*

PETALS - Find an image of your state flower here http://www.statesymbolsusa.org/Lists/state_flowers.html. Complete this chart to see if your state flower follows Fibonacci, too. [*answers will vary*]

State:	Fibonacci? Y / N Explain:
Flower:	
Number of petals:	

LEAVES follow Fibonacci's sequence as they alternate their way up stems.

4. Review leaf arrangements with Fibonacci here: http://www.maths.surrey.ac.uk/hosted-sites/R.Knott/Fibonacci/fibnat.html#plants. The mathematical patterns are cool, but how does a Fibonacci-based arrangement of leaves on the stem actually help the plant?
   Leaves grow so that lower ones are never fully covered by upper ones. This allows sunlight and rain to reach each leaf along the stem.
5. Click through the photos http://theobrominated.blogspot.com/2011/06/she-loves-me-she-loves-me-not.html
   Review the discussion of leaf and petal spirals with some advanced biological terms.
   a. Define *monostichous – the upper leaf is directly above the lower leaf; there is one complete turn (360^0) of the stem between each leaf*
   b. Define *distichous – the upper and lower leaves alternate by 180^0; there is ½ a turn of the stem between each leaf*
6. Collect at least one stem with leaves. Draw and label your sample below. Labeling can include plant name, location, scale/actual dimensions, monostichous or distichous. [*answers will vary*]